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Chiral Corrections to Matrix Elements of Twist-2 Operators

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Abstract

We compute the leading non-analytic contributions of the form $m_q \log m_q$ to matrix elements of twist-2 operators in the nucleon and pion using effective field theory. Previously omitted one-loop contributions that are related to tree-level matrix elements by chiral symmetry are included.

I. INTRODUCTION

Deep-Inelastic-Scattering (DIS) from nucleon targets has provided a wealth of information about the nature of strong interactions and the structure of the nucleon. From the initial discovery of partons in the 1970's, DIS continues to provide ever more precise measurements of the parton-distribution functions (PDFs). It is highly desirable to make a direct connection between the experimental data and the now well-established theory of strong interactions, QCD. A rigorous and model-independent comparison will be accomplished by performing high-statistics unquenched or partially-quenched [1] lattice-QCD calculations [2]. Presently, lattice calculations cannot be performed with the physical values of the light quark masses, m_q, ($m_u \sim 5$ MeV, $m_d \sim 10$ MeV) and extrapolations from the lattice masses, that produce a pion of mass $m_\pi^{\rm latt.} \sim 500$ MeV, to the physical values must be performed. Of course, such extrapolations require knowledge of the m_q-dependence of the matrix element of interest. Recently hadronic models, such as the Cloudy Bag Model (CBM), have been used to motivate explicit forms for the m_q-dependence of forward matrix elements of the non-singlet twist-2 operators that contribute to DIS from the nucleon [3,4]. In addition, these models have been used to connect lattice calculations to other properties of the nucleon, such as electromagnetic form factors [5].

It is well established that one can determine the m_q-dependence of hadronic observables by performing a systematic expansion about the chiral limit [6–9]. In fact, extensive work has been accomplished in understanding the properties and interactions of the low-lying mesons and baryons in both two-flavour and three-flavour QCD, such as the magnetic moments [10], the electric form factors [11], the axial matrix elements [12], and the polarizabilities of the nucleons and other octet baryons [13], to name just a few, and the analogous quantities in hadrons containing heavy quarks [14]. In addition, there has been an extensive effort during the last decade to include multi-nucleon systems in this framework [15]. In this work, we include twist-2 operators in the chiral lagrangian, and compute the leading non-analytic contributions of the form $m_q \log m_q$ to their matrix elements.

The pion fields are introduced into the low-energy effective field theory (EFT), chiral perturbation theory (χPT), through the Σ-field,

$$\Sigma = \exp\left(\frac{2iM}{f}\right) \quad , \quad M = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}\,\tau^\alpha\,\pi^\alpha \quad . \tag{1}$$

with $f = 132\,\mathrm{MeV}$. $\Sigma = \xi^2$ transforms as $\Sigma \to L\Sigma R^\dagger = L\xi U^\dagger\,U\xi R^\dagger$ under $SU(2)_L \otimes SU(2)_R$ chiral transformations. In order to construct an EFT with well-defined power-counting, the nucleons are treated as heavy fields in the Heavy-Baryon formulation of Jenkins and Manohar [7], and transform as $N_v \to UN_v$ (the subscript v denotes the four-velocity of the nucleon) under chiral transformations (for reviews see Ref. [8,9]). Below the chiral symmetry breaking scale Λ_χ, S-matrix elements can be expanded in derivatives and in m_q. The naive size of the matrix element of an operator with n_1 creation operators for heavy nucleons, n_2 annihilation operators for heavy nucleons, n_3 derivatives, n_4 light quark mass matrices, n_5 powers of the nucleon four-velocity v, n_6 Σ-field operators and n_7 $\Sigma^\dagger$-field operators, is

$$f^2\Lambda_\chi^2\left(\frac{\bar{N}_v}{f\sqrt{\Lambda_\chi}}\right)^{n_1}\left(\frac{N_v}{f\sqrt{\Lambda_\chi}}\right)^{n_2}\left(\frac{\partial}{\Lambda_\chi}\right)^{n_3}\left(\frac{m_q}{\Lambda_\chi}\right)^{n_4}(v)^{n_5}(\Sigma)^{n_6}\left(\Sigma^\dagger\right)^{n_7} \quad , \tag{2}$$

which should be considered merely as a guide. The strong interactions between pions and nucleons at leading order in the chiral expansion arise from a lagrange density of the form

$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left[\ \partial^\mu\Sigma\ \partial_\mu\Sigma^\dagger\ \right]\ +\ \lambda\ \mathrm{Tr}\left[\ m_q\Sigma^\dagger\ +\ \mathrm{h.c.}\ \right]$$
$$+\ \overline{N}_v\ iv\cdot DN_v\ +\ 2g_A\overline{N}_v\ S\cdot\mathcal{A}\ N_v \quad , \tag{3}$$

where $D_\mu = \partial_\mu + \mathcal{V}_\mu$ is the chiral-covariant derivative with $\mathcal{V}_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right)$ the pion vector field, $g_A = 1.25$ is the axial-vector coupling constant, S^μ is the covariant spin-operator defined in Ref. [7], $\mathcal{A}_\mu = \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right)$ is the axial-vector pion field, and λ is a parameter that provides the leading order contribution to the pion mass.

II. NON-SINGLET, TWIST-2 OPERATORS

In this section we will focus on forward matrix elements of the non-singlet twist-2 operators

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n} = \frac{1}{n!}\ \overline{q}\ \tau^a\ \gamma_{\{\mu_1}\left(i\ \overleftrightarrow{D}_{\mu_2}\right)\ ...\left(i\ \overleftrightarrow{D}_{\mu_n\}}\right)\ q\ -\ \mathrm{traces} \quad , \tag{4}$$

where the $\{...\}$ denotes symmetrization on all Lorentz indices. Once the transformation properties of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ under $SU(2)_L\otimes SU(2)_R$ have been established, the operators in the pion-nucleon EFT that reproduce matrix elements of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ can be constructed, and further, the EFT can be used to perform a model-independent calculation of the low-momentum contributions to its matrix element. In order to determine its transformation properties it is convenient to write it in terms of left-handed and right-handed quark fields,

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n} = \mathcal{O}^{(n),a}_{L,\ \mu_1\mu_2\ ...\mu_n}\ +\ \mathcal{O}^{(n),a}_{R,\ \mu_1\mu_2\ ...\mu_n} \tag{5}$$

with

$$\mathcal{O}^{(n),a}_{L,\ \mu_1\mu_2\ ...\mu_n} = \frac{1}{n!}\ \overline{q}_L\ \tau^a_L\ \gamma_{\{\mu_1}\left(i\ \overleftrightarrow{D}_{\mu_2}\right)\ ...\left(i\ \overleftrightarrow{D}_{\mu_n\}}\right)\ q_L\ -\ \mathrm{traces}$$
$$\mathcal{O}^{(n),a}_{R,\ \mu_1\mu_2\ ...\mu_n} = \frac{1}{n!}\ \overline{q}_R\ \tau^a_R\ \gamma_{\{\mu_1}\left(i\ \overleftrightarrow{D}_{\mu_2}\right)\ ...\left(i\ \overleftrightarrow{D}_{\mu_n\}}\right)\ q_R\ -\ \mathrm{traces} \quad , \tag{6}$$

from which it is clear that $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ transforms as $(3,1)\oplus(1,3)$ under $SU(2)_L\otimes SU(2)_R$. For bookkeeping purposes we have introduced the flavour matrices τ^a_L and τ^a_R that are taken to transform as $\tau^a_L \rightarrow L\tau^a_L L^\dagger$ and $\tau^a_R \rightarrow R\tau^a_R R^\dagger$ under $SU(2)_L\otimes SU(2)_R$.

If we are interested in DIS from pions, or in pion loop contributions to DIS from the nucleon we require the matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ in the pion. The power-counting rules of eq. (2) dictate that such matrix elements will be dominated by operators involving the least number of derivatives and insertions of m_q, but can have an arbitrary number of insertions of the Σ and $\Sigma^\dagger$ fields. Operators that can contribute are of the form

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n} \rightarrow a^{(n)}\,(i)^n\,\frac{f^2}{4}\left(\frac{1}{\Lambda_\chi}\right)^{n-1}\ \mathrm{Tr}\left[\Sigma^\dagger\tau^a\overrightarrow{\partial}_{\mu_1}...\overrightarrow{\partial}_{\mu_n}\Sigma\ +\ \Sigma\tau^a\overrightarrow{\partial}_{\mu_1}...\overrightarrow{\partial}_{\mu_n}\Sigma^\dagger\right]\ -\ \mathrm{traces}$$
$$= a^{(n)}2\,(i)^n\left(\frac{1}{\Lambda_\chi}\right)^{n-1}\ i\varepsilon^{\alpha a\beta}\ \pi^\alpha\overrightarrow{\partial}_{\mu_1}...\overrightarrow{\partial}_{\mu_n}\pi^\beta\ -\ \mathrm{traces}\ +\ \mathcal{O}\left(\pi^4\right) \quad . \tag{7}$$

With the exception of $n = 1$, the coefficients $a^{(n)}$ are unknown and must be determined elsewhere. $\mathcal{O}^{(1)}_{\mu_1}$ is the isovector charge operator from which we deduce that $a^{(1)} = +1$. In addition to the operators of eq. (7), there are also operators of the form

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n} \rightarrow a^{(n)}\ (i)^n\ \frac{f^2}{4}\left(\frac{1}{\Lambda_\chi}\right)^{n-1}\ \ \mathrm{Tr}\left[\Sigma^\dagger\tau^a\overrightarrow{\partial}_{\{\mu_1}\ldots\overrightarrow{\partial}_{\mu_k}\Sigma\overrightarrow{\partial}_{\mu_k}\ldots\overrightarrow{\partial}_{\mu_p}\Sigma^\dagger\ldots..\overrightarrow{\partial}_{\mu_r}\ldots\overrightarrow{\partial}_{\mu_n\}}\Sigma\right.$$
$$\left. +\ \Sigma\leftrightarrow\Sigma^\dagger\right]\ -\ \mathrm{traces}\quad , \tag{8}$$

that must be considered. However, these operators do not contribute to single-pion forward matrix elements at tree-level, and only give interactions between three or more pions. In addition, the symmetry of the Lorentz indices and the tracelessness of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n}$ means that these operators do not contribute to single-pion forward matrix elements even at one-loop level. Therefore the diagrams shown in fig. 1 give the leading non-analytic contributions

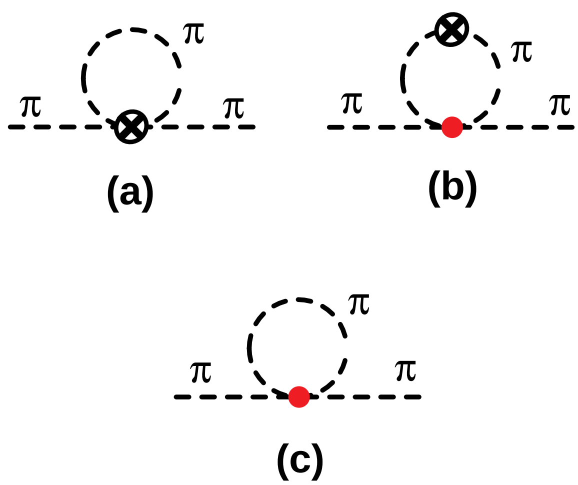

FIG. 1. *The pion loop diagrams that give the leading non-analytic contributions to the matrix element of* $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n}$ *between single-pion states. The crossed circle denotes an insertion of an operator from eq. (7), arising directly from the twist-2 operator. The smaller solid circle denotes an insertion of a leading order strong-interaction vertex from eq. (3). Diagrams (a) and (b) are vertex corrections while diagram (c) denotes wavefunction renormalization.*

to the matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n}$ between single-pion states. After a straightforward calculation, one finds that the forward matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n}$ between an initial pion with isospin index α and momentum q_μ, and a final pion with isospin index β vanishes for n-even, and is

$$\mathcal{M} = i\ 4\ a^{(n)}\ \left(\frac{1}{\Lambda_\chi}\right)^{n-1}\ \left[1 - \frac{1-\delta^{n1}}{8\pi^2 f^2}m_\pi^2\log\left(\frac{m_\pi^2}{\Lambda_\chi^2}\right)\ +\ \ldots\right]\ \varepsilon^{\alpha\beta a}\ q_{\mu_1}\ldots q_{\mu_n}\ -\ \mathrm{traces}\quad , \tag{9}$$

for n-odd, where the ellipses denote terms that are analytic in m_q, or are higher order in the chiral expansion. The factor of $1 - \delta^{n1}$ in the sub-leading contribution ensures that the $n = 1$ isospin-charge is not renormalized at loop-level.

The leading order operator contributing to the matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ between nucleon states is

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n} \ \rightarrow \ A^{(n)}\ v_{\mu_1}v_{\mu_2}...v_{\mu_n}\ \overline{N}_v\ \tau^a_{\xi+}\ N_v\ -\ \mathrm{traces} \quad , \tag{10}$$

where operators involving more derivatives or insertions of m_q are suppressed by powers of Λ_χ, and we have defined $\tau^a_{\xi\pm}$ to be

$$\tau^a_{\xi\pm} = \frac{1}{2}\left(\ \xi\tau^a\xi^\dagger \pm \xi^\dagger\tau^a\xi\ \right) \quad . \tag{11}$$

The coefficients $A^{(n)}$ must be determined elsewhere, except for $A^{(1)} = +1$ which corresponds to the isospin charge operator. In addition to the tree-level contribution to the nucleon forward matrix element, there are vertices involving an even number of pion fields. The diagrams shown in fig. 2 give the leading non-analytic corrections to the forward matrix

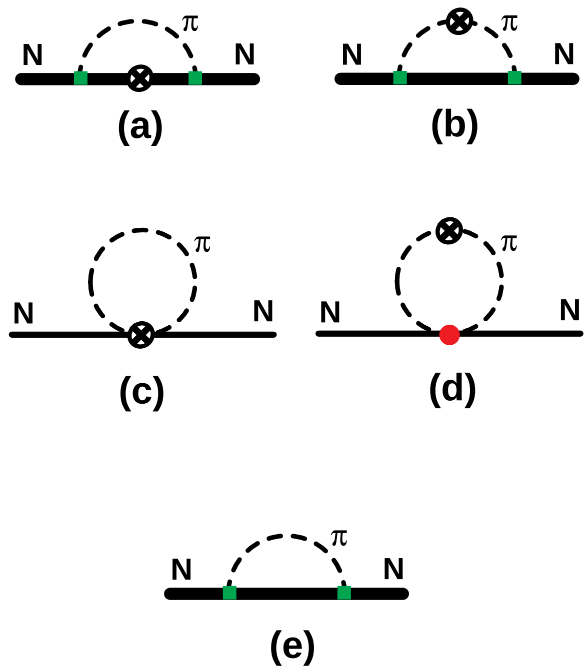

FIG. 2. *The pion loop diagrams that give the leading non-analytic contributions to the matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ between single-nucleon states. The crossed circle denotes an insertion of an operator from eq. (7) or eq. (10), arising directly from the twist-2 operator. The smaller solid circle denotes an insertion of the strong two-pion-nucleon interaction from the nucleon kinetic energy term in eq. (3), while the square denotes an insertion of the axial-vector interaction $\propto g_A$. Diagrams (a)-(d) are vertex corrections while diagram (e) denotes nucleon wavefunction renormalization.*

element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ between single-nucleon states. A straightforward computation gives

$$\mathcal{M} = A^{(n)}\ v_{\mu_1}v_{\mu_2}...v_{\mu_n}\ \overline{U}_v\tau^a U_v\ \left[\ 1\ -\ \left(3g_A^2+1\right)\frac{1-\delta^{n1}}{8\pi^2 f^2}m_\pi^2\log\left(\frac{m_\pi^2}{\Lambda_\chi^2}\right)\ \right] \quad , \tag{12}$$

where U_v is the nucleon spinor, and we have only shown the non-analytic part of the sub-leading contribution. The non-analytic contributions to the $n = 1$ matrix element vanish as the nucleon isospin charge is not renormalized. Our result in eq. (12) differs from previous computations [3,4] primarily due to our inclusion of the two-pion-nucleon interaction associated with the tree-level matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n}$, but also in the numerical coefficient of the g_A^2 contribution. In Ref. [3] only the contribution from diagram (b) of fig. 2 is computed and correctly found to scale as $m_q^{\frac{n+1}{2}} \log m_q$. However, this diagram is only part of the complete result for $n = 1$ (required by charge conservation) and is sub-dominant for $n > 1$. In Ref. [4], this was corrected somewhat by the appearance of $m_q \log m_q$ contributions for all values of n, however, the g_A independent contributions were omitted.

It is worth keeping in mind the relative size of non-analytic terms from loop-diagrams compared with the analytic contributions from both loop-diagrams and local counterterms. The complete set of local operators with a single insertion of m_q that contribute to the matrix element of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n}$ is

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n} \rightarrow \Big(b_1(\mu)\ \overline{N}\ \{\tau^a_{\xi+}, \chi_+\}\ N\ +\ b_2(\mu)\ \overline{N}\ \left[\tau^a_{\xi+}, \chi_-\right]\ N\ +\ b_3(\mu)\ \mathrm{Tr}\left[\chi_+\right]\ \overline{N}\ \tau^a_{\xi+}\ N \\ +\ b_4(\mu)\ \overline{N}\ \{\tau^a_{\xi-}, \chi_-\}\ N\ +\ b_5(\mu)\ \overline{N}\ \left[\tau^a_{\xi-}, \chi_+\right]\ N\ \Big)\ v_{\mu_1} v_{\mu_2} \dots v_{\mu_n} \quad , \tag{13}$$

where $\xi_\pm = \frac{1}{2}\left(\xi^\dagger m_q \xi^\dagger \pm \xi m_q^\dagger \xi\right)$. In general, the constants $b_i(\mu)$ are renormalization scale dependent, and must be determined experimentally. Only the operators with coefficients b_1 and b_3 contribute to forward matrix elements in the nucleon. The operators with coefficients b_2 and b_5 have at least one additional pion associated with them, while b_4 has at least two additional pions associated with it. The m_q-dependence of the matrix elements of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n}$ shown in eqs. (9), (12) and (13) has the form

$$\mathcal{M} \sim \alpha\ +\ \beta m_q \log\left(m_q/\mu\right)\ +\ \gamma(\mu) m_q\ +\ \dots\ , \tag{14}$$

where the μ-dependence of $\gamma(\mu)$ is precisely equal and opposite that of the β-term, leaving an expression that is explicitly μ-independent. While the contributions from the β-term formally dominate the sub-leading contributions in the chiral limit when $\mu = \Lambda_\chi$, there are well-known examples where such terms are numerically smaller than the sub-leading analytic contributions, terms analogous to the γ-term, for the physical values of m_q. The pion-charge radius $\langle r_\pi^2 \rangle$ is such an example, where for the physical values of the pion mass (and kaon mass in $SU(3)$), the contribution from the $\alpha_9(\mu)$ counterterm (evaluated at $\mu = \Lambda_\chi$) is twice that of the non-analytic loop contributions of the form $\log m_q$. Therefore, while the terms we have computed in eq. (12) are the formally dominant sub-leading contributions in the chiral limit they may not dominate the sub-leading contribution for physical values of m_q due to the terms shown in eq. (13).

At relatively low momentum scales, $\sim$ 300 MeV, there can be large contributions from loop diagrams involving the Δ's. The formal construction and phenomenology of an EFT with dynamical Δ's (or any resonance) has been studied extensively [7–14,16]. If all diagrams with the Δ-resonance as an intermediate state are included then one can consistently take $\mu \sim \Lambda_\chi$, and capture the dominant infrared behavior of the theory [7,8]. Matrix elements of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n}$ between Δ states are described, at leading order, by

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n} \ \rightarrow\ C^{(n)}\ v_{\mu_1}v_{\mu_2}\ldots v_{\mu_n}\ \overline{\Delta}^{\alpha}_{v}\ \tau^{a}_{\xi+}\ \Delta_{\alpha,v}\ -\ \text{traces}$$
$$+\ D^{(n)}\ \frac{1}{n!} v_{\{\mu_1}v_{\mu_2}\ldots v_{\mu_{n-2}}\ \overline{\Delta}_{\mu_{n-1},v}\ \tau^{a}_{\xi+}\ \Delta_{\mu_n\},v}\ -\ \text{traces} \quad , \tag{15}$$

where $C^{(1)} = -3$ by normalization of the isospin charge operator, and $D^{(1)} = 0$ simply because of the number of available Lorentz indices. These operators will contribute to matrix elements between nucleon states through loop diagrams. The leading order strong interactions between the N's, Δ's and π's are described by a lagrange density of the form

$$\mathcal{L}^{\Delta} = g_{N\Delta}\ \left[\overline{\Delta}^{\alpha,ijk}\ \mathcal{A}^{l}_{\alpha,k}\ N_j\ \epsilon_{il}\ +\ \text{h.c.}\right] \quad , \tag{16}$$

where the coupling is $g_{N\Delta} \sim 1.8$, from the observed width for $\Delta \rightarrow N\pi$ [8]. The loop diagrams shown in fig. 3 gives rise to a forward matrix element between single-nucleon

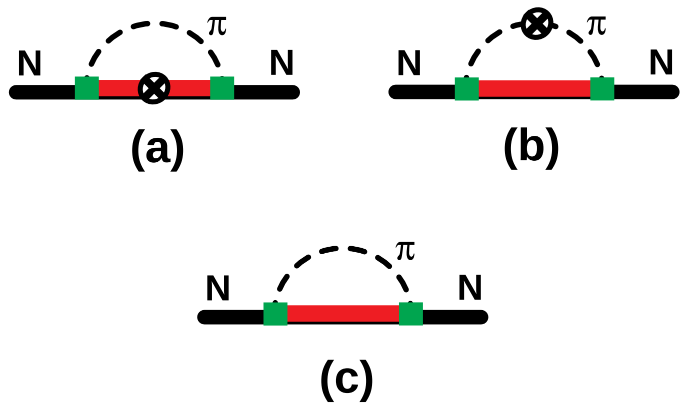

FIG. 3. *One-loop diagrams with Δ intermediate states that contribute to the matrix elements of $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n}$ between single-nucleon states. The thick solid line inside the loop denotes a Δ propagator, while the dashed line denotes a pion propagator. The crossed circle denotes an insertion of an operator from eq. (7) or eq. (15), arising directly from the twist-2 operator, and the square denotes an insertion of the strong $N\Delta\pi$ interaction $\propto g_{N\Delta}$. Diagrams (a) and (b) are vertex corrections while diagram (c) denotes nucleon wavefunction renormalization.*

states of

$$\mathcal{M} = -\frac{g^2_{N\Delta}}{4\pi^2 f^2}\ J_1\left(\Delta M, m_\pi\right)\ \overline{U}_v \tau^a U_v\ \left[\ A^{(n)}\ +\ \frac{5}{9}C^{(n)}\ -\ \frac{5}{27}D^{(n)}\ +\ \frac{2}{3}\delta^{n1}\ \right]$$
$$\left(\ v_{\mu_1}v_{\mu_2}\ldots v_{\mu_n}\ \ -\ \text{traces}\ \right) \quad , \tag{17}$$

where $\Delta M = M_\Delta - M_N$ is the Δ-N mass difference. The function J_1 is

$$J_1\left(\Delta, m\right)\ =\left(m^2 - 2\Delta^2\right)\log\left(\frac{m^2}{\Lambda^2_\chi}\right)\ +\ 2\Delta\sqrt{\Delta^2 - m^2}\log\left(\frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}\right) \quad , \tag{18}$$

where we have renormalized at the scale $\mu = \Lambda_\chi$. In the limit that $\Delta \rightarrow 0$, J_1 contains a chiral logarithm, as is clear from eq. (18), while in the limit of large Δ only terms analytic in m survive, as required by the decoupling of the Δ [8]. The reason that such contributions must be included in order to sensibly renormalize the theory at $\mu = \Lambda_\chi$ is that the scale for the m_q-dependence from these diagrams is set by ΔM and not by Λ_χ, and therefore a naive estimate of the size of counterterms in the theory without the explicit Δ-fields is set by ΔM and not Λ_χ. Without resorting to hadronic models, one is unable to make statements about $C^{(n)}$ or $D^{(n)}$, and they must be determined elsewhere. It is interesting to note that $N\Delta$ transition operators induced by $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ involve either a spin-operator S^μ or additional pion fields. Neither type of operator contributes to nucleon matrix elements at one-loop level. Our result in eq. (17) disagrees with the result of Ref. [3], as they computed only the contribution from fig. 3 (b), where $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ ...\mu_n}$ is inserted into the pion propagator.

III. SINGLET, TWIST-2 OPERATORS

In this section we consider matrix elements of singlet twist-2 operators, ${}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ ...\mu_n}$ with $j = q, g$ for the quark and gluonic operators, of the form

$$\begin{aligned} {}^{(S)}\mathcal{O}^{q(n)}_{\mu_1\mu_2\ ...\mu_n} &= \frac{1}{n!}\ \overline{q}\ \gamma_{\{\mu_1}\left(i\ \overleftrightarrow{D}_{\mu_2}\right)\ ...\left(i\ \overleftrightarrow{D}_{\mu_n\}}\right)\ q\ -\ \text{traces} \\ {}^{(S)}\mathcal{O}^{g(n)}_{\mu_1\mu_2\ ...\mu_n} &= \frac{1}{n!}\ G^a_{\alpha\{\mu_1}\left(i\ \overleftrightarrow{D}_{\mu_2}\right)\ ...\left(i\ \overleftrightarrow{D}_{\mu_{n-1}}\right)\ G^{a,\alpha}_{\mu_n\}}\ -\ \text{traces}\quad . \end{aligned} \tag{19}$$

and it is clear that the ${}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ ...\mu_n}$ transform as $(1,1)$ under $SU(2)_L \otimes SU(2)_R$.

The matrix element of ${}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ ...\mu_n}$ in the pion will be described at leading order by a lagrange density of the form

$$\begin{aligned} {}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ ...\mu_n} &\rightarrow \bar{a}^{(n)}_j\ (i)^n\ \frac{f^2}{4}\left(\frac{1}{\Lambda_\chi}\right)^{n-1}\ \text{Tr}\left[\Sigma^\dagger\ \overrightarrow{\partial}_{\mu_1}...\overrightarrow{\partial}_{\mu_n}\Sigma\ +\ \Sigma\overrightarrow{\partial}_{\mu_1}...\overrightarrow{\partial}_{\mu_n}\Sigma^\dagger\right]\ -\ \text{traces} \\ &= \bar{a}^{(n)}_j 2\ (i)^n\left(\frac{1}{\Lambda_\chi}\right)^{n-1}\ \pi^\alpha\overrightarrow{\partial}_{\mu_1}...\overrightarrow{\partial}_{\mu_n}\pi^\alpha\ -\ \text{traces}\ +\ \mathcal{O}\left(\pi^4\right)\quad , \end{aligned} \tag{20}$$

where the $\bar{a}^{(n)}_j$ are coefficients that must be determined elsewhere, and depend upon the particular singlet operator under consideration. A calculation of one-loop diagrams analogous to those shown in fig. 1 give rise to a matrix element between pions with isospin indices α and β, at next-to-leading order, of

$$\mathcal{M}_j = 4\ \bar{a}^{(n)}_j\left(\frac{1}{\Lambda_\chi}\right)^{n-1}\left[1 - \frac{1}{8\pi^2 f^2}m_\pi^2\log\left(\frac{m_\pi^2}{\Lambda_\chi^2}\right)\ +\ ...\ \right]\ \delta^{\alpha\beta}\ q_{\mu_1}...q_{\mu_n}\ -\ \text{traces}\quad , \tag{21}$$

for n-even, while the matrix element for n-odd vanishes. The non-analytic correction is the same for both quark and gluonic operators.

In contrast to eq. (10), the leading-order operator contributing to the matrix element of a singlet operator ${}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ ...\mu_n}$ between nucleon states is

$${}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ ...\mu_n}\ \rightarrow\ \bar{A}^{(n)}_j\ v_{\mu_1}v_{\mu_2}...v_{\mu_n}\ \overline{N}_v\ N_v\ -\ \text{traces}\quad , \tag{22}$$

where operators involving more derivatives or insertions of m_q are suppressed by powers of Λ_χ. Since ${}^{(S)}\mathcal{O}^{q(1)}_\mu$ is the baryon number operator and ${}^{(S)}\mathcal{O}^{g(n)}_{\mu_1\mu_2\ ...\mu_n}$ vanishes for $n < 2$, we have that $\bar{A}_q^{(1)} = +3$ and $\bar{A}_g^{(1)} = 0$. Only some of the one-loop diagrams in fig. 2 contribute to matrix elements of the singlet operators. Diagrams (c) and (d) of fig. 2 are absent while diagram (b) can only contribute at higher orders. Further, the contribution from the vertex diagram, diagram (a), is exactly canceled by the contribution from wavefunction renormalization, diagram (e). Therefore, the singlet matrix elements in the nucleon do not receive any non-analytic corrections of the form $m_q \log m_q$ from nucleon intermediate states [1]. However, there are contributions from Δ intermediate states. The leading order matrix elements involving the Δ are described by the operators

$$\begin{aligned} {}^{(S)}\mathcal{O}^{j(n)}_{\mu_1\mu_2\ \ldots\mu_n} \ \rightarrow\ & \bar{C}_j^{(n)}\ v_{\mu_1}v_{\mu_2}\ldots v_{\mu_n}\ \overline{\Delta}_v^\alpha\ \Delta_{\alpha,v}\ -\ \text{traces} \\ & +\ \bar{D}_j^{(n)}\ \frac{1}{n!} v_{\{\mu_1}v_{\mu_2}\ldots v_{\mu_{n-2}}\ \overline{\Delta}_{\mu_{n-1},v}\ \Delta_{\mu_n\},v}\ -\ \text{traces}\quad , \end{aligned} \tag{23}$$

with $\bar{C}_q^{(1)} = -3$, $\bar{C}_g^{(1)} = 0$, $\bar{D}_q^{(1)} = 0$, and $\bar{D}_g^{(1)} = 0$. These operators contribute through loop-diagrams shown in fig. 3. Diagram (b) of fig. 3 contributes only at higher orders in the chiral expansion, and we find a contribution to the nucleon matrix element of

$$\begin{aligned} \mathcal{M}_j = -\frac{g_{N\Delta}^2}{4\pi^2 f^2}\ J_1\left(\Delta M, m_\pi\right)\ \overline{U}_v U_v\ & \left[\ \bar{A}_j^{(n)}\ +\ \bar{C}_j^{(n)}\ -\ \frac{1}{3}\bar{D}_j^{(n)}\ \right] \\ \left(\ v_{\mu_1}v_{\mu_2}\ldots v_{\mu_n}\ \ -\ \text{traces}\ \right) & \quad . \end{aligned} \tag{24}$$

As required, corrections to the matrix element of the baryon number operator, ${}^{(S)}\mathcal{O}^{q(1)}_\mu$, vanish.

IV. CONCLUSIONS

We have computed the leading non-analytic contributions of the form $m_q \log m_q$ to matrix elements of twist-2 operators that arise in deep-inelastic scattering. A previously omitted contribution to the matrix elements of non-singlet operators that is independent of g_A and related by chiral symmetry to the tree-level vertex is identified. Our results will aid in the extrapolation of unquenched lattice calculations of single-nucleon matrix elements of twist-2 operators from the quark masses used on the lattice to their physical values.

The work we have presented here can be straightforwardly extended to off-forward matrix elements of the twist-2 operators, i.e. those in which there is a non-zero momentum transfer to the hadronic system from the twist-2 operator. Such matrix elements have received significant amount of attention during the past few years, as one can define off-forward parton distributions as a simple extension of the parton model (for an overview see Ref. [17]). In addition, deeply-virtual-Compton-scattering (DVCS) has been extensively explored as a means to measure such distributions. The effective field theory construction we have employed in this work will allow for a description of these matrix elements in the low-momentum regime.

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[1] The loop corrections are the same as those contributing to the nucleon mass, for which there are no terms of the form $m_q \log m_q$.

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